A sparse blow-up lemma

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The Regularity Method has been used to solve many problems in extremal graph theory in the last fifteen years. It consists of three parts: the Regularity Lemma, the Counting Lemma and the Blow-up Lemma. Also in the last fifteen years, there has been increasing interest in extremal questions relative to sparse graphs - in particular, relative to random or to pseudorandom graphs. Naturally, one would like to apply the Regularity Method to answer such questions, but unfortunately none of its three components is applicable to sparse graphs.

A Sparse Regularity Lemma (due to Kohayakawa and to Rödl) applicable to random or pseudorandom graphs has existed for almost two decades, and more recently a version due to Scott is applicable to all sparse graphs. But there is no sparse version of the Counting Lemma or Blow-up Lemma applicable to general sparse graphs (the obvious statements are false). Recently sparse versions of the Counting Lemma were proved for random graphs (by Balogh, Morris and Samotij, by Saxton and Thomason, and by Conlon, Gowers, Samotij and Schacht) and pseudorandom graphs (by Conlon, Fox and Zhao). But in order to address problems involving large or spanning subgraphs of random or pseudorandom graphs, a sparse Blow-up Lemma is required.

We (with Böttcher, Hán, Kohayakawa and Person) have proved versions of the Blow-up Lemma applicable to relatively dense subgraphs of sparse random or pseudorandom graphs. In this talk I will explain what exactly a Blow-up Lemma is, how it must be modified for the sparse setting (and what exactly that means), and what applications a sparse Blow-up Lemma has.

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