

Title: Minimum cuts in planar graphs: sampling, counting, and connectivity

Speaker: Ivona Bezáková (Rochester Institute of Technology)

Abstract:

We will discuss three minimum cut problems in weighted planar graphs: (1) counting and sampling minimum source-sink cuts, (2) counting and sampling “contiguous” minimum single-source-multi-sink cuts, and (3) finding a minimum “simple” single-source-multi-sink cut.

All three problems have applications in computer vision and medical imaging where cuts can be used, for example, to isolate an object from its background. A sampling algorithm provides means for statistical analysis of the cuts and connectivity requirements such as contiguity and simplicity are useful when one wants to prevent splitting the object into several disconnected parts.

We will sketch an $O(n^2)$ algorithm counting all minimum source-sink cuts in weighted planar graphs and an $O(n^3)$ algorithm counting all contiguous minimum single-source-multi-sink cuts, where n is the number of vertices and contiguity roughly corresponds to “corner-connectivity” on the grid. In both cases, after having completed the counting part, subsequent sampling is very fast: a uniformly random cut can be produced in additional linear time. We will also outline an $O(n^5)$ algorithm that finds a minimum simple cut - that is, a cut where the cut set induces a connected subgraph.

The counting algorithms reduce the problem to the problem of counting a different type of cuts in an unweighted planar directed acyclic graph (these cuts can also be thought of as maximal antichains in the corresponding partially ordered set). These cuts correspond to certain cycles in the planar dual graph and we employ dynamic programming to count them.

The simple cut algorithm takes a completely different approach as it cannot rely on the well-known maximum-flow-minimum-cut duality. Instead, we analyze the combinatorial structure of simple cuts, guaranteeing the existence of a simple cut that does not enter a certain “forbidden” region in the plane. Then, a new dynamic programming approach allows us to form this cut by connecting dual paths along the boundary of the “forbidden” region.

The talk is based on joint works with Adam Friedlander and Zachary Langley.