The Phase Transition for the Giant Component in Random Hypergraphs

Oliver Cooley
Technische Universität Graz

A classical result in random graph theory is the fact that the Erdős-Rényi random graph model $G(n,p)$ exhibits a phase transition for the existence of a giant component when $p \sim \frac{1}{n}$. More precisely, given $\varepsilon > 0$, the largest component in $G(n, \frac{1-\varepsilon}{n})$ has size $O(\log n)$ with high probability, while $G(n, \frac{1+\varepsilon}{n})$ has a unique "giant component" with $\Theta(n)$ vertices with high probability. Recently Krivelevich and Sudakov [3] gave a new and very elegant proof of this fact.

We examine the generalisation of the method of Krivelevich and Sudakov to random k-uniform hypergraphs, in which we may generalise the notion of connectedness for graphs to j-connectivity for each $1 \leq j \leq k-1$. For vertex connectivity ($j = 1$), the phase transition has already been proved [4, 2, 1]. In this talk we present a proof of a phase transition result for general j and k.

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References

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