On generalized Ramsey numbers of Erdős and Rogers

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In a graph G, a set $S \subseteq V(G)$ is called *s-independent* if the subgraph of G induced by S does not contain K_s. Let the *s-independence number* of G, denoted by $\alpha_s(G)$, be the size of the largest s-independent set in G. (Hence, in particular, $\alpha_2(G)$ is the ordinary independence number.) The classical *Ramsey number* $R(t,u)$ can be defined in this language as the smallest integer n such that $\alpha_2(G) \geq u$ for every K_t-free graph G of order n. A more general problem results by replacing the ordinary independence number by the s-independence number. Following this approach, in 1962 Erdős and Rogers introduced the function

$$f_{s,t}(n) = \min\{\alpha_s(G) : G \text{ is a } K_t\text{-free graph of order } n\}.$$

In this talk, we present some old and recent developments concerning this function. In particular, we partially confirm an old conjecture of Erdős by showing that $\lim_{n\to\infty} \frac{f_{s+1,s+2}(n)}{f_{s,s+2}(n)} = \infty$ for any $s \geq 4$ (joint work with John Retter and Vojta Rödl). Furthermore, we discuss some extensions for hypergraphs (joint work with Dhruv Mubayi).