MCMC sampling colourings and independent sets of $G(n, d/n)$ near uniqueness threshold

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Abstract

Sampling from *Gibbs distribution* is a central problem in computer science as well as in statistical physics. In this work we focus on the *k-colouring model* and the *hard-core model* with fugacity λ when the underlying graph is an instance of Erdős-Rényi random graph $G(n, p)$, where $p = d/n$ and d is fixed.

We use the *Markov Chain Monte Carlo* method for sampling from the aforementioned distributions. In particular, we consider *Glauber (block) dynamics*. We show a dramatic improvement on the bounds for *rapid mixing* in terms of the number of colours and the fugacity for the corresponding models. For both models the bounds we get are only within small constant factors from the conjectured ones by the statistical physicists.

We use *Path Coupling* to show rapid mixing. For k and λ in the range of our interest the technical challenge is to cope with the high degree vertices, i.e. vertices of degree much larger than the expected degree d. The usual approach to this problem is to consider block updates rather than single vertex updates for the Markov chain. Taking appropriately defined blocks the effect of high degree vertices somehow diminishes. However, devising such a construction of blocks is a *highly non trivial* task.

We develop for a first time a weighting schema for the paths of the underlying graph. Vertices which belong to "light" paths, only, can be placed at the boundaries of the blocks. Then the tree-like local structure of $G(n, d/n)$ allows the construction of simple structured blocks.

Interestingly enough, the weighting schema captures so well the desired properties of the blocks that we do not even need to argue explicitly on *spatial mixing of Gibbs distribution* for the range of k, λ we consider. However, we do believe that our approach has further consequences as far as correlation decay is regarded. Thus, apart from the standard path coupling analysis, the main bulk of the analysis we use is of probabilistic nature.

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