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Title: A relative Szemerédi theorem

Abstract:

Green and Tao proved that the primes contain arbitrarily long arithmetic progressions. The proof has two parts. The first part is a relative Szemerédi theorem which says that any subset of a pseudorandom set of integers of positive relative density contains long arithmetic progressions, where a set is pseudorandom if it satisfies two conditions, the linear forms condition and the correlation condition. The second part is in finding a pseudorandom set in which the primes have positive relative density.

In this talk, I will discuss recent joint work with David Conlon and Yufei Zhao in which we give a simple proof for a strengthening of the relative Szemerédi theorem, showing that a weak linear forms condition is sufficient for the theorem to hold. By removing the correlation condition, our strengthened version can be applied to give a relative Szemerédi theorem for k-term arithmetic progressions in pseudorandom subsets of Z_N of density $N^{-c(k)}$. It also simplifies the deduction of the Green-Tao theorem by removing the need for certain number theoretic estimates in the second part of their proof.

The key component in our proof is an extension of the regularity method to sparse pseudorandom hypergraphs, which we believe to be interesting in its own right. From this we derive a relativized hypergraph removal lemma. This is a strengthening of an earlier theorem used by Tao in his proof that the Gaussian primes contain arbitrarily shaped constellations and, by standard arguments, allows us to deduce the relative Szemerédi theorem.