Title: Sperner’s Theorem and a Problem of Erdős-Katona-Kleitman.

Name and Affiliation Wenying Gan, Ph.D candidate in UCLA

Abstract: A central result in extremal set theory is the celebrated theorem of Sperner from 1928, which gives the size of the largest family of subsets of \([n]\) not containing a 2-chain \(F_1 \subsetneq F_2\). Erdős extended this theorem to determine the largest family without a \(k\)-chain \(F_1 \subsetneq F_2 \subsetneq \ldots \subsetneq F_k\). Erdős and Katona, followed by Kleitman, asked how many chains must appear in families with sizes larger than the corresponding extremal bounds.

In 1966, Kleitman resolved this question for 2-chains, showing that the number of such chains is minimized by taking sets as close to the middle level as possible. Moreover, he conjectured the extremal families were the same for \(k\)-chains, for all \(k\). In this talk we present a first progress on this problem, verifying Kleitman’s conjecture for the families whose size is at most the size of the \(k+1\) middle levels. We also characterize all extremal configurations.

Joint work with Shagnik Das and Benny Sudakov.