Distances in random Apollonian networks

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The construction of random Apollonian networks (RAN) originates from the problem of Apollonian circle packing. Starting from a planar embedding of the complete graph on four vertices, in each step pick an inner face of the graph uniformly at random, place a node inside it and connect it with all three of its neighbors. The result is a RAN in two dimensions. Zhang et al. generalized the construction to arbitrary dimensions in a natural way.

RANs exhibit the properties that real-life networks posses. In particular, we focus on the small world property which refers to the phenomena that the shortest paths between vertices are considerably smaller than the size of the graph. When studying shortest paths three quantities of interest are the minimal path between two uniformly chosen vertices (hopcount), the flooding time of a fixed vertex (maximal hopcount) and the diameter (maximal flooding time) of the graph.

We show that in arbitrary d dimensions all three quantities asymptotically scale in probability as $c_d \log n$. The constant c_d is determined for all three cases and a central limit theorem is proved for the minimal path. The result extends previous work of Albenque and Marckert on the shortest path in two dimensions and the work of Frieze and Tsourakakis, who showed an upper bound for the diameter also in two dimensions.

The proof makes use of the nice hierarchical structure of RANs, a direct relation to continuous time branching processes and uses some basic renewal theory.