The diameters of two random graph models

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Broutin and Devroye [1] developed a technique that uses Cramér's Theorem on large deviations to compute the height of a class of weighted random trees. We adapt this technique to estimate the diameters of two random graph models, defined below. Our results hold asymptotically almost surely (a.a.s.) as the number of vertices grow.

The first model is called a *random apollonian network*. Consider the following iterative construction of a random planar triangulation. Start with a triangle embedded in the plane. In each step, choose a bounded face uniformly at random, add a vertex inside that face and join it to the vertices of the face. After $n-3$ steps, we obtain a random triangulated plane graph with n vertices. We prove that a.a.s. the diameter of the resulting graph is asymptotic to $c \log n$, where $c \approx 1.668$ is the solution of an explicit equation.

The second model is called a *random surfer tree model*. Given $p \in (0, 1]$ we incrementally build a random digraph in which all vertices have a unique out-neighbour. Let $X_1, X_2, \ldots$ be independent geometric random variables with parameter p. Start with a single vertex v_0 with a directed loop. In step $i \geq 1$ add a new vertex v_i, choose a random vertex u in the present graph, and perform a walk of length X_i starting from u. If the last vertex of the walk is w, add a directed edge from v_i to w. The resulting random graph is a special case of a model proposed for the Web graph. For $p \geq 0.21$ we determine the asymptotic value of the diameter of the underlying undirected tree, and for $p < 0.21$ we provide lower and upper bounds that are logarithmic in the number of vertices.

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References

[1] N. Broutin and L. Devroye. Large deviations for the weighted height of an extended class of trees, *Algorithmica* 46 (2006), 271–297.