Arboricity and spanning-tree packing in random graphs
with an application to load balancing

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Abstract

We study the arboricity $A$ and the maximum number $T$ of edge-disjoint spanning trees of the Erdős-Rényi random graph $\mathcal{G}(n, p)$. For all $p(n) \in [0, 1]$, we show that, with high probability, $T$ is precisely the minimum between $\delta$ and $\lfloor m/(n-1) \rfloor$, where $\delta$ is the smallest degree of the graph and $m$ denotes the number of edges. Moreover, we explicitly determine a sharp threshold value for $p$ such that: above this threshold, $T$ equals $\lfloor m/(n-1) \rfloor$ and $A$ equals $\lceil m/(n-1) \rceil$; and below this threshold, $T$ equals $\delta$, and we give a two-value concentration result for the arboricity $A$ in that range. Finally, we include a stronger version of these results in the context of the random graph process where the edges are sequentially added one by one. A direct application of our result gives a sharp threshold for the maximum load being at most $k$ in the two-choice load balancing problem, where $k \to \infty$. This research is joint work with Pu Gao and Cristiane M. Sato.