Zero-one laws for random distance graphs with vertices in $\{-1,0,1\}^n$

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In 1969 Glebskii Y. et al. in [1] proved the zero-one law for Erdős–Rényi random graphs. Later S. Shelah and J. Spencer expanded the class of random graphs that follow the zero-one law (see [2]). Zero-one laws for random distance graphs have been considered for the first time by M. Zhukovskii (see [3, 4]).

Let us define the model of a random distance graph, generalizing the model from [3, 4]. Let G_n be the graph (V_n, E_n), where

$$V_n = \{\mathbf{v} = (v^1, \dots, v^n) : v^i \in \{-1,0,1\}, \quad |\{i \in \{1,\dots,n\} : v^i = 1\}| = a,$$
$$|\{i \in \{1,\dots,n\} : v^i = -1\}| = b, \quad |\{i \in \{1,\dots,n\} : v^i = 0\}| = d = n-a-b\},$$

$$E_n = \{\{\mathbf{u},\mathbf{v}\} \in V_n \times V_n : (\mathbf{u},\mathbf{v}) = c\},$$

where $(\mathbf{u},\mathbf{v})$ is the Euclidean scalar product. The random distance graph with vertices in $\{-1,0,1\}^n$ is the probabilistic space $\mathcal{G}(G_n,p) = (\Omega_{G_n}, \mathcal{F}_{G_n}, \mathsf{P}_{G_n,p})$, where

$$\Omega_{G_n} = \{G = (V,E) : V = V_n, E \subseteq E_n\},$$

$$\mathcal{F}_{G_n} = 2^{\Omega_{G_n}}, \quad \mathsf{P}_{G_n,p}(G) = p^{|E|}(1-p)^{|E_n|-|E|}.$$

We prove the following results about the zero-one law for $\mathcal{G}(G_n,p)$.

Theorem 1. *Let $a-b = o(n), c = o(a-b)$, $a = \Theta(n)$, $d(n) \to \infty, n \to \infty$, and for every $m \in \mathbb{N}$, there exists $n_0 \in \mathbb{N}$ such that, for every $n > n_0$, numbers $a(n)-b(n)$ and $c(n)$ are divisible by m. Then the random distance graph $\mathcal{G}(G_n,p)$ follows the zero-one law.*

Theorem 2. *Let $a(n)-b(n) = \alpha n, c(n) = \alpha^2 n$, $\alpha \in \mathbb{Q}$, $0 < \alpha < 1$, $d(n) \to \infty, n \to \infty$. Then there exists a subsequence $\{\mathcal{G}(G_{n_i},p)\}_{i\in\mathbb{N}}$, following the zero-one law.*

We also give some more general and complicated conditions guaranteeing the existence of a subsequence, following the zero-one law, and find some cases, when the random distance graph doesn't follow the zero-one law.

References

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