The chromatic numbers of random subgraphs of distance graphs

L. Bogolyubskiy, A. Gusev, M. Pyaderkin, A. Raigorodskii

Our talk is concerned with the classical Nelson–Hadwiger problem on finding the chromatic numbers of distance graphs in $\mathbb{R}^n$. We mainly consider a class of graphs $G(n, r, s) = (V(n, r), E(n, r, s))$ defined as follows:

$$V(n, r) = \{ x = (x_1, \ldots, x_n) : x_i \in \{0, 1\}, \; x_1 + \ldots + x_n = r \},$$

$$E(n, r, s) = \{ \{ x, y \} : (x, y) = s \},$$

where $(x, y)$ is the Euclidean scalar product. In particular, recently the chromatic number of $G(n, 3, 1)$ was found by J. Balog, A. Kostochka, A. Raigorodskii (see [1]).

We study the random graphs $G(G(n, r, s), p)$ whose edges are chosen independently from the set $E(n, r, s)$ each with probability $p$. We find concentration results for the independence numbers of such graphs and bounds for their chromatic numbers. We also study some algorithmic aspects of the above-mentioned questions.

References


\footnote{Moscow State University, Mechanics and Mathematics Faculty, Department of Mathematical Statistics and Random Processes; Moscow Institute of Physics and Technology, Department of Discrete Mathematics}