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The typical structure of sparse K_{r+1}-free graphs

Two central topics of study in combinatorics are the so-called evolution of random graphs, introduced by the seminal work of Erdős and Rényi, and the family of H-free graphs, that is, graphs which do not contain a subgraph isomorphic to a given (usually small) graph H. A widely studied problem that lies at the interface of these two areas is that of determining how the structure of a typical H-free graph with n vertices and m edges changes as m grows from 0 to $\mathrm{ex}(n, H)$. We resolve this problem in the case when H is a clique, extending a classical result of Kolaitis, Prömel, and Rothschild. In particular, we prove that for every $r \geqslant 2$, the is an explicit constant θ_r such that, letting

$$m_r = \theta_r n^{2-\frac{2}{r+2}} (\log n)^{1/\left[\binom{r+1}{2}-1\right]},$$

the following holds for every positive constant ε. If $m \geqslant (1+\varepsilon)m_r$, then almost all K_{r+1}-free n-vertex graphs with m edges are r-partite, whereas if $n \ll m \leqslant (1-\varepsilon)m_r$, then almost all of them are not r-partite. (This result in the case $r = 2$ was obtained ten years ago by Osthus, Prömel, and Taraz).

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