Let $\{G_t\}_{t \geq 0}$ be the random graph process ($G_0$ is edgeless and $G_t$ is obtained by adding a uniformly distributed new edge to $G_{t-1}$), and let $\tau_k$ denote the minimum time $t$ such that the $k$-core of $G_t$ (its unique maximal subgraph with minimum degree at least $k$) is nonempty. For any fixed $k \geq 3$ the $k$-core is known to emerge via a discontinuous phase transition, where at time $t = \tau_k$ its size jumps from 0 to linear in the number of vertices with high probability. It is believed that for any $k \geq 3$ the core is Hamiltonian upon creation w.h.p., and Bollobás, Cooper, Fenner and Frieze further conjectured that it in fact admits $\lfloor (k-1)/2 \rfloor$ edge-disjoint Hamilton cycles. However, even the asymptotic threshold for Hamiltonicity of the $k$-core in $G(n,p)$ was unknown for any $k$.

We show here that for any fixed $k \geq 15$ the $k$-core of $G_t$ is w.h.p. Hamiltonian for all $t \geq \tau_k$, i.e., immediately as the $k$-core appears and indefinitely afterwards. Moreover, we prove that for large enough fixed $k$ the $k$-core contains $\lfloor (k-3)/2 \rfloor$ edge-disjoint Hamilton cycles w.h.p. for all $t \geq \tau_k$.

Joint work with M. Krivelevich and E. Lubetzky