TITLE: Cores of random graphs are born Hamiltonian

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ABSTRACT:

Let $\{G_t\}_{t\geq 0}$ be the random graph process (G_0 is edgeless and G_t is obtained by adding a uniformly distributed new edge to G_{t-1}), and let τ_k denote the minimum time t such that the k-core of G_t (its unique maximal subgraph with minimum degree at least k) is nonempty. For any fixed $k \geq 3$ the k-core is known to emerge via a discontinuous phase transition, where at time $t = \tau_k$ its size jumps from 0 to linear in the number of vertices with high probability. It is believed that for any $k \geq 3$ the core is Hamiltonian upon creation w.h.p., and Bollobás, Cooper, Fenner and Frieze further conjectured that it in fact admits $\lfloor (k-1)/2 \rfloor$ edge-disjoint Hamilton cycles. However, even the asymptotic threshold for Hamiltonicity of the k-core in $\mathcal{G}(n,p)$ was unknown for any k.

We show here that for any fixed $k \geq 15$ the k-core of G_t is w.h.p. Hamiltonian for all $t \geq \tau_k$, i.e., immediately as the k-core appears and indefinitely afterwards. Moreover, we prove that for large enough fixed k the k-core contains $\lfloor (k-3)/2 \rfloor$ edge-disjoint Hamilton cycles w.h.p. for all $t \geq \tau_k$.

Joint work with M. Krivelevich and E. Lubetzky