The time of bootstrap percolation with dense initial sets

Andrew J. Uzzell
Uppsala University

Abstract

In r-neighbor bootstrap percolation on the vertex set of a graph G, vertices are initially infected independently with some probability p. At each time step, the infected set expands by infecting all uninfected vertices that have at least r infected neighbors. We study the distribution of the time at which all vertices become infected. Given $d \geq r \geq 2$ and $t = t(n) = o\big((\log n / \log \log n)^{1/(d-r+1)}\big)$, we prove a sharp threshold result for the probability that percolation occurs by time t in r-neighbor bootstrap percolation on the d-dimensional discrete torus $\mathbb{T}_n^d$. Moreover, we show that for certain ranges of $p = p(n)$, the time at which percolation occurs is concentrated either on a single value or on two consecutive values. We also prove corresponding results for the modified d-neighbor rule and for the subcritical case $d + 1 \leq r \leq 2d$.

This is joint work with B. Bollobás, C. Holmgren, and P. J. Smith.