PROJECTIONS OF FRACTAL PERCOLATIONS IN HIGHER DIMENSIONS

LAJOS VÁGÓ

This is a joint work with Károly Simon. We consider the fractal percolations which are one of the most well studied examples of random Cantor sets. Rams and Simon [1] studied the projections of fractal percolation sets on the plane. We extend the scope of their theorem and generalize it to higher dimensions.

Fractal percolations, or Mandelbrot percolations on the plane are defined in the following way: Fix an integer $M \geqslant 2$ and probabilities $0 < p_{i,j} < 1$, $i, j = 1, \dots, M$. Then partition the unit square K into M^2 congruent squares of side length $1/M$, let us call these $K_{i,j}$, $i, j = 1, \dots, M$. Then retain all small squares $K_{i,j}$ with probability $p_{i,j}$ independently from each other, or discard them otherwise. Repeat this procedure in the retained squares ad infinitum to finally get a random set E called fractal percolation.

It is well known that if all the probabilities $p_{i,j}$ are greater than $1/M$ and smaller than a critical probability p_c, then E has Hausdorff dimension greater than 1 with positive probability, and conditioned on non-emptiness it is totally disconnected. However, in [1] the authors gave a rather complicated technical condition under which the orthogonal projection of E (which is a random dust) in all directions contains some interval, conditioned on E being nonempty.

In our work, we generalize this result to higher dimensions, i.e. for fixed $d \geqslant 2$ and $d > k \geqslant 1$ we consider orthogonal projections of d-dimensional fractal percolation to all k-dimensional planes at once, and obtain the same result as in dimension 2. We adapt the same random inverse Markov operator as in [1]. Also some geometrical issues has to be handled in the higher dimensional case.

References

[1] Rams, M. and Simon, K.: *Projections of fractal percolations*, *http://arxiv.org/abs/1306.3844*, (2013), to appear in Ergodic Theory and Dynamical Systems.

Lajos Vágó, Department of Stochastics, Institute of Mathematics, Technical University of Budapest, 1521 Budapest, P.O.Box 91, Hungary

E-mail address: vagolala@math.bme.hu